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Research on the Evolution and Core Structure of Low Mass Stars

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Abstract: This article studies the complete evolution process of low-quality stars from main sequence stars to red giants, and then to white dwarfs, and analyzes in detail the physical characteristics of the stellar core. By applying the stellar structure equation, energy generation rate formula, and degeneracy pressure formula, this paper derives the evolution laws of temperature and density of the stellar core over time, and explores the influence of these physical properties on different stages of stellar evolution. Through detailed analysis of the stellar core at various stages, this article reveals the physical evolution mechanism of low mass stars during their lifecycle.

Keywords: Main sequence star; White dwarfs; Subgiants; Red giants; Planetary nebulae

1. Introduction

Low mass stars (with masses between 0.08 and 2 times that of the Sun) are very common in the universe, accounting for over 70% of all stars in the Milky Way galaxy. Due to the smaller mass and lower core temperature of these stars, the rate of nuclear fusion reaction is slower, resulting in a much longer evolution time for low mass stars than for high mass stars. Studying the evolution process of low-quality stars not only helps us understand the different stages of stellar life cycles, but also provides key clues to the evolutionary history of stars in the Milky Way galaxy.

Low mass stars undergo several major stages in their lifecycle: main sequence stars, subgiants, red giants, planetary nebulae, and ultimately evolve into white dwarfs. Each stage has unique physical processes and kernel changes. This article will quantitatively analyze the changes in the core structure and physical state of these stars at different stages through detailed mathematical models and physical derivations.

2. Main sequence star stage

2.1 Hydrogen fusion in main sequence stars

In the main sequence stage, the main source of energy for low mass stars comes from hydrogen fusion in their cores. This process is carried out through proton proton chain reaction (p-p chain reaction), which is the most important nuclear reaction mechanism in low mass stars. The overall equation for this reaction process is:

$$4\mathrm{H} \rightarrow \mathrm{He} + 2e^{+} + 2\nu_{e} + \gamma$$

In this reaction, four hydrogen nuclei (protons) are transformed into a helium nucleus through a series of intermediate reactions, releasing two positrons, two neutrinos, and gamma rays simultaneously. The mass difference Δm in this process is converted into energy through Einstein's mass energy equation:

$$\Delta E = \Delta m c^{2}$$

Among them, Δm is the difference in mass before and after the reaction, and c is the speed of light. This energy is transported to the surface of stars through radiation and convection, and ultimately released into the universe as light and heat.

The reaction rate of hydrogen fusion is closely related to the core temperature T, Assuming a stellar mass of M, the relationship between reaction rate and temperature T can be expressed as:

$$\epsilon_{pp} \propto T^{4}$$

Among them, ϵ_{pp} is the energy generated per second per unit mass. Due to the strong dependence of reaction rate on temperature, the temperature of the stellar core has a decisive impact on its energy output.

2.2 Kernel Physical Characteristics

The structure of a star is described by four fundamental equations, These equations describe the mass distribution, pressure distribution, energy generation and transmission inside stars:

(1) Conservation of mass equation:

$$\frac{dM_r}{dr} = 4\pi r^2\rho$$

(2) Static equilibrium equation:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

(3) Energy generation equation:

$$\frac{dL_r}{dr} = 4\pi r^2\rho\,\epsilon$$

(4) Energy transfer equation:

$$\frac{dT}{dr} = -\frac{3\kappa\rho L_r}{16\pi acT^3r^2}$$

Among them, M_r is the mass at radius r, P is the pressure, ρ is the density, L_r is the luminosity, ϵ is the energy production rate, κ is the opacity of the star, a is the radiation constant, and c is the speed of light.

These equations determine the core temperature and density of a star. By numerical calculation, the core temperature of low-quality stars in the main sequence stage is approximately 10^7 K, and the core density is about tens of grams per cubic centimeter.

3. Sub giant and Red Giant Stage

3.1 Core Contraction during the Superstar Stage

As the main sequence stage approaches its end, the hydrogen in the star's core gradually runs out, and the hydrogen fusion rate significantly decreases. Due to the core no longer producing enough energy to balance gravity, it begins to contract, and as it contracts, its temperature and density gradually increase.

The ideal gas state equation continues to apply at this stage:

$$P = \frac{\rho k_B T}{\mu m_H}$$

Among them, P is pressure, k_B is Boltzmann constant, μ is average molecular weight, and m_H is the mass of hydrogen atoms.

As the core shrinks, the density ρ increases, the temperature T also rises, leading to more intense combustion of the hydrogen shell. At this point, nuclear fusion in the hydrogen shell begins to play a dominant role, and the expansion of the outer layer of the star causes it to enter the subgiant stage.

3.2 Helium fusion in the red giant stage

When the core temperature rises to about 100 million K, the triple alpha reaction of helium begins:

$3\alpha \to C + \gamma$

The dependence of reaction rate on temperature is:

$\epsilon_{3\alpha} \propto T^{40}$

Such high temperature sensitivity means that once the core temperature reaches a critical value, helium fusion reactions will rapidly and violently unfold within the core. At this stage, the star expands to a huge volume and enters the red giant stage. The luminosity of red giants significantly increases, while their surface temperature is relatively low, resulting in a typical red color.

3.3 Kernel Physical Characteristics

The core of the red giant stage is supported by electron degeneracy pressure, This pressure is different from the pressure of ordinary gases, and it originates from the degeneracy pressure of Fermi gas:

$$P_e = \frac{\left(3\pi^2\right)^{2/3}\hbar^2}{5m_e}\left(\frac{Z\rho}{Am_H}\right)^{5/3}$$

Among them, h is Planck's constant, m_e is electron mass, Z is nuclear charge number, and A is mass number. At this point, the density of the core can reach millions of grams per cubic centimeter, with a temperature of approximately 10^8 K.

The core density of stars in the red giant stage can reach millions of grams per cubic centimeter, with a core temperature of approximately 10^8 K. The degeneracy of the core leads to the further evolution direction of the star, that is, entering the final planetary nebula and white dwarf stage.

4. Planetary Nebula and White Dwarf Stage

4.1 Formation of Planetary Nebulae

In the later stage of the red giant stage, due to the end of helium fusion, the star's core no longer has enough energy to support the outer material. At this point, the material in the outer layer of the star begins to be ejected, forming planetary nebulae. These expanding gas shells leave the core at a relatively high speed and eventually expose the high-temperature and dense stellar core, which is the prototype of white dwarfs. The expansion rate of planetary nebulae can be estimated using the following formula:

$$v_{\exp} = \sqrt{\frac{2GM}{R}}$$

Among them, $v_{\exp}$is the expansion velocity, G is the gravitational constant, R is the radius of the stellar shell and M is the stellar mass. At this stage, the material in the stellar shell is ejected into space due to the decrease in gravity and the increase in radiation pressure, forming beautiful planetary nebulae. The expansion rate of planetary nebulae typically ranges from 10 to 50 kilometers per second.

4.2 Core Structure of White Dwarfs

The formation of planetary nebulae marks the final stage of stellar evolution, namely the white dwarf stage. A white dwarf is the remnant of a star after its death, with its core composed of high-density degenerate matter, mainly composed of carbon and oxygen. The core of a white dwarf is supported by electron degeneracy pressure instead of traditional thermal compression.

There is an inverse relationship between the mass and radius of a white dwarf, meaning that the heavier the white dwarf, the smaller its radius. The mass of a white dwarf has a theoretical maximum, known as the Chandrasekhar limit, which is given by the following formula:

$$M_{Ch} = \frac{5.83\hbar c}{Gm_H^2} \approx 1.4M_{\odot}$$

Among them, $\hbar$ is Planck's constant, c is the speed of light, G is the gravitational constant, m_H is the mass of a proton, and $M_{\odot}$ is the mass of the sun. The Chandrasekhar limit represents the maximum possible mass of a white dwarf, beyond which the star will no longer be stable and may collapse into a neutron star or black hole.

The density of white dwarfs is extremely high, typically ranging from 10^6 to 10^9

grams per cubic centimeter. Although its volume is relatively small, its mass is close to that of the Sun. Due to the lack of nuclear fuel, white dwarfs can only gradually cool down through thermal radiation and eventually become a cooled black dwarf.

4.3 Kernel Physical Characteristics

In the white dwarf stage, the physical properties of the stellar core are dominated by degenerate pressure. Due to the fact that the electron degeneracy pressure is independent of the temperature of the star, white dwarfs can maintain extremely high densities even as the temperature gradually decreases. The volume of a white dwarf can be estimated using the following formula:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{3\hbar^2}{GM^2}\right)^{1/3}$$

This formula represents that the volume of a white dwarf decreases with increasing mass, given a given mass M. This phenomenon indicates that the radius of dwarf stars decreases as their mass increases, ultimately forming a highly compressed celestial body.

The temperature of a white dwarf may reach tens of thousands of K during its early formation, but over time, the white dwarf will gradually cool down, and the final temperature may drop to several thousand K or even lower. A cooled white dwarf may evolve into a black dwarf, however, due to the limited age of the universe, no black dwarf has been discovered so far.

5. Conclusion

The evolution process of low-quality stars exhibits significant changes in their internal physical properties such as temperature and density. Starting from hydrogen fusion in the main sequence star stage, the core temperature gradually increases; In the red giant stage, the core undergoes helium fusion and is ultimately supported by degenerate pressure to form a white dwarf. Through the derivation of the stellar structure equation, energy generation formula, and degeneracy pressure formula, this article reveals the core characteristics and physical change mechanisms of stars at different stages of evolution.

The core of a star in the main sequence stage mainly releases energy through hydrogen fusion to maintain the stable structure of the star; As hydrogen is depleted, the star enters the subgiant and red giant stages, during which the helium fusion in the core dominates the release of energy and causes significant expansion of the outer layers of the star. In the later stage of the red giant stage, the degeneracy pressure of the core prevents further collapse, ultimately forming a stable white dwarf. This study provides a quantitative physical model and derivation process for understanding the evolution and changes in core structure of low mass stars.

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