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Insights on Fostering Students' Innovation Consciousness

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Abstract: The 2011 and 2022 editions of compulsory curriculum standards underscore the importance of fostering innovation consciousness among students their development. Based on our teaching practice in recent years, we explore how students' innovation awareness can be fostered, focusing on six aspects.

Keywords: Teaching practice; Innovation consciousness

Innovation consciousness typically refers to students' proactive attempts to discover and raise meaningful mathematical problems based on daily life, natural phenomena, or in scientific contexts. According to the 2011 edition of compulsory curriculum standards, to meet the requirements of talent cultivation with the evolution of times, mathematics curriculum should emphasize on cultivating innovative thinking among students throughout their education(1), starting from the stage of compulsory education. In the 2022 edition of compulsory curriculum standard, innovation consciousness is considered a part of core mathematical education, encouraging students to observe the real world and develop their curiosity, imagination, and innovation from the perspective of mathematics. Innovation consciousness allows them to develop independent thinking, willingness to challenge and inquire, a scientific mindset, and a rational spirit(2).

1. Expanding thinking by suggesting multiple solutions to one problem

Question 1: Divide an equilateral triangle into three congruent parts.

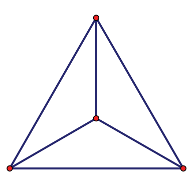

Fig. 1

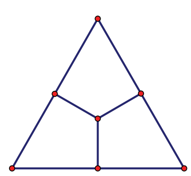

Fig. 2

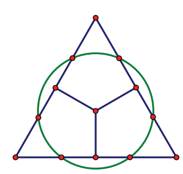

Fig. 3

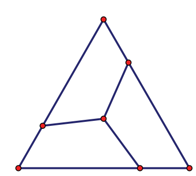

Fig. 4

When asked to divide an equilateral triangle into congruent parts, students provided distinct solutions. Fig. 1 shows the division suggested by some students.

Another student drew the diagram shown in Fig. 2, dividing an equilateral triangle into three identical quadrangles, which was applauded by all students and teachers. Since the students are relatively familiar with triangles, but relatively unfamiliar with quadrangles, it was not easy to come up with this answer. Are there other answers? Yes, there are countless ways to divide an equilateral triangle.

An innovative solution to this question is to draw a circle with a radius greater than the inradius (radius of the incircle) and smaller than the circumradius (radius of the circumcircle), centered at the centroid of the equilateral triangle, (Fig. 3). As shown in Fig. 3, the circle intersects each side of the triangle at two points. The corresponding three points can be chosen to divide the triangle into three congruent parts, as shown in Fig. 4. Moreover, because the radius of the circle can vary, there can be countless ways to solve this problem. These demonstrations enhanced students' knowledge, which, in turn, expanded their thinking to some extent.

2. Thinking exercise through classified discussion

Question 2: As shown in Fig. 5, given that the point P is on the parabola $y=-x^2+4x+2$; point Q is on the x-axis; and the quadrilateral with vertices D (2, 6), E (4, 2), P and Q is a parallelogram, determine the coordinates of the point P.

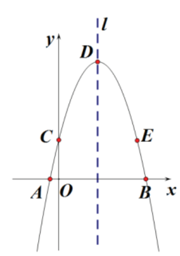

Fig. 5

Analysis: Using the central point coordinate formula of line segment, the quadrangle with vertices D, E, P, and Q was a parallelogram; thus, we set up three equation sets through classified discussion to answer the question; the solutions that failed to satisfy the conditions were eliminated.

Given the point Q (n, 0), point P (m , $-m^2+4m+2$)

Since the quadrangle with vertices D, E, P, and Q is a parallelogram, the question can be discussed in three situations.

1. The vertices D and E are opposite to each other in the parallelogram, as shown in Fig. 6.

Given that the diagonals of a parallelogram bisect each other into two equal segments, the central point of the line segment DE was also the central point of line segment PQ. Accordingly, the equation was set as follows:

$$\begin{cases} \dfrac{2+4}{2}=\dfrac{m+n}{2} & (1) \\ \dfrac{6+2}{2}=\dfrac{-m^2+4m+2}{2} & (2) \end{cases}$$

By organizing Equation (2), we get $m^2-4m+6=0$

Because $\Delta=b^2-4ac=(-4)^2-4\times1\times6=-8<0$,

The equation has no real root; thus, this case does not exist.

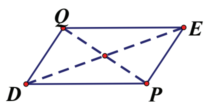

Fig. 6

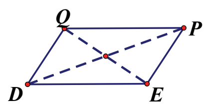

Fig. 7

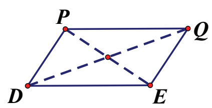

Fig. 8

2. D and P are two opposite vertices in the parallelogram, as shown in Fig. 7.

The central point of line segment DP is also the central point of line segment EQ. Accordingly, the equation set as follows:

$$\begin{cases} \dfrac{n+4}{2}=\dfrac{m+2}{2} & (1) \\ \dfrac{2+0}{2}=\dfrac{-m^2+4m+2+6}{2} & (2) \end{cases}$$

After organizing Equation (2), we get $m^2-4m-6=0$, resulting in the equation $m=2\pm\sqrt{10}$

$-m^2+4m+2=-(m^2-4m)+2=-6+2=-4$,

Hence, the coordinates of point P are ($2+\sqrt{10}$, -4)and($2-\sqrt{10}$, -4).

3. D and Q are opposite vertices in the parallelogram, as shown in Fig. 8.

The central point of line segment DQ is also the central point of line segment EP. Thus, the following equation was set:

$$\begin{cases} \dfrac{n+2}{2}=\dfrac{m+4}{2} & (1) \\ \dfrac{6+0}{2}=\dfrac{-m^2+4m+2+2}{2} & (2) \end{cases}$$

By organizing Equation (2), we get the equation $m^2-4m+2=0$. The answer is $m=2\pm\sqrt{2}$.

$-m^2+4m+2=-(m^2-4m)+2=-(-2)+2=4$,

Hence, the coordinates of point P were ($2+\sqrt{2}$, 4)and($2-\sqrt{2}$, 4)

In summary, the coordinates of point P were ($2+\sqrt{10}$, -4), ($2-\sqrt{10}$, -4), ($2+\sqrt{2}$, 4), and($2-\sqrt{2}$, 4).

Notes: Based on the problem statement, Q (n, 0) and P (m, $-m^2+4m+2$), and using the central point coordinate formula for line segment, the problem is discussed in three scenarios and worked out with the corresponding equation set without referring to the original geometric figure. Thus, the geometric problem changed into an algebraic problem, which decreased the difficulty level of logical thinking. The equations set were all quadratic equations in two variables, except for Equation (2), which was a quadratic equation in one variable and was solved smoothly. In case there was no solution, it was inferred that this case does not exist. After the x-coordinate was worked out, the y-coordinate could be easily worked out with substitution as a whole.

3. Improving thinking through funny mathematics

A problem was encountered on the blog of Dr. Peng Xicheng from Central China Normal University.

Question 3: Someone drives from location A to location B at a speed of 30 km/h, and returns from B to A at a very fast speed. Can the average speed of the whole trip reach 60 km/h?

How can students answer this problem? The problem was presented while offering a reward for the correct answer. The students were asked to hand in their answers the next morning. The next day, some students were seen discussing the problem, including those with a poor foundation and little interest in mathematics. Upon entering the classroom, some students explained their answers to the teacher on the blackboard.

Most students showed interest in the problem. Their specific answers are summarized as follows:

3.1 Wrong answers

Answer 1: When one returns from B to A at a very fast speed, the average speed will surely reach 60 km/h.

Comment: This answer is based on the imagination of some students, lacking mathematical analysis, and was marked as a subjective error.

Answer 2, 60×2=120 (km), 120-30=90 (km)

Answer 2: When one returns from B to A at a speed exceeding 90 km/h, then the average speed of the whole trip can reach 60 km/h.

Comment: This is the opinion of some students, which is a confusion of concepts, that is, average speed ≠ mean speed. Average speed = total distance ÷ total time, rather than (speed of the departing trip + speed of the returning trip) ÷ 2. The This was also marked as an error for the same reason of as for A answer 1.

3.2 Correct answers

Hypothesis methods:

Answer 1: Assuming the average speed to be 60 km/h, and the distance between A and B to be 600 km, the time taken for the departing trip was calculated as 600÷30=20 h. If the average speed was 60 km/h, then the time of the round trip should be 1200÷60=20 h. Unless one returns to B instantly upon arriving at A, the average speed in the whole trip cannot reach 60 km/h.

Comment: The hypothesis method is typically used for solving mathematics problems. A suitable hypothesis can make abstract problems concrete, thus reducing the difficulty level and making it easier to solve the given problem.

Answer 2: Suppose the time taken to go from A to B is m hours, and the distance between A and B is L km, then L÷m=30, 2L÷60=m, the average speed of the round trip is 60 km/h, and it takes m h. Yet, the departing trip has taken m h; then, there is no time for the returning trip.

Comment: This answer is also based on the hypothesis method; however, because the assumed time and distance were both expressed as a letter, the thinking appears somewhat abstract, making the answer more rigorous and logical.

Setting up the equations:

Answer 3: Suppose the distance between A and B is S km, and the time taken to return from B to A is t h:

$$\frac{2S}{\frac{S}{30}+t}=60,\ 2S=60\times\left(\frac{30}{S}+t\right),$$

$$2S=2S+60t,\ t=0,$$

Thus, the average speed of the whole trip cannot reach 60 km/h.

Comment: Here, two unknown variables were set up. The fractional equation was set up according to the formula: average speed = total distance ÷ total time. The time taken for returning from B to A was calculated to be 0, which is practically impossible; hence, we reached the conclusion that the average speed of the whole trip cannot possibly reach 60 km/h.

Answer 4: Suppose the speed of returning trip is x km/h, and the distance between A and B is m km. The time of departing trip and the time of returning trip = the time of round trip, $\frac{m}{30}+\frac{m}{x}=\frac{2m}{60}$, which can be simplified as $\frac{1}{x}=0$. Obviously, this is impossible.

Comment: Here, two unknown variables are set up, but unlike in Answer 3, their relation of equality is different, and so the equation. The simplified equation considers the time when the value of the fractional equation will be zero. The numerator is 1, so whatever value x holds, the value of the fractional equation cannot be 0; thus, the answer should be “No”.

3.3 Some reflections

This problem is quite funny; it can solidify students’ understanding of the formula for average speed and can be worked out using primary school arithmetic or using fractional equations taught in Junior 2. The obtained solution should be tested and verified to check whether it is consistent with real-life scenarios. It can also deepen students’ understanding of the problem when the value of a fractional equation is 0, which is not completely the same as the answer obtained by setting up a fractional equation as introduced in textbooks. It is indeed a tricky problem.

This challenge with a reward aroused students’ interest in mathematics, as every student began thinking deeply and sought help from family and friends or online resources, highlightings the charm of mathematics. The next day, the students explained their answers and thoughts, while other students nodded and applauded their answers. Their work paid off, and they received a small reward from the teacher. The students who gave wrong answers were also encouraged. In doing so, their innovation in learning mathematics was fostered.

4. Clarify thinking through mathematics games

Inspired by the CCTV program “Is It Real?” we believe that mathematical knowledge can also be obtained from games. Especially in review classes, knowledge points that can easily confuse students are given in the form of "True" or "False" questions. The students are asked to provide reasons for a "True" answer; otherwise, they are asked to name counter-examples to explain why the answer is "False." The rules of the game are explained to the students, and then, they are asked to guess and discuss and finally answer the question. One of the students is selected as a host, who should be smart and clever and be able to arouse their interest and atmosphere, encouraging wonderful performance among the students. We present an example to explain this.

Question 4: Please answer whether the follow statements are true or false:

1. A quadrilateral with diagonals that bisect each other is a parallelogram.
2. A quadrilateral with equal diagonals is a parallelogram.
3. A parallelogram with equal diagonals is a rectangle.
4. A quadrilateral with diagonals that are equal and bisect each other is a parallelogram.
5. A quadrilateral with perpendicular diagonals is a rhombus.
6. A parallelogram with perpendicular diagonals is a rhombus.
7. A quadrilateral with perpendicular, bisecting, and equal diagonals is a square.

The difficulty lies in the determination of special parallelograms, especially from the perspective of diagonals. The host shows a problem each time, allowing the students to discuss, who can directly speak out their answer. It should be analyzed if it is "True"; otherwise, if the answer is "False, " counter-examples should be named. The statements 1, 3, and 6 were answered by the students as "True." In Statement 4, they determined it as a parallelogram based on the fact of bisecting diagonals, and determined it as a rectangle based on the fact of equal diagonals. In Statement 7, some students said it was a rectangle, a rhombus, or a square. In statements 2 and 5, the students named counter-examples, as shown in Fig. 9.

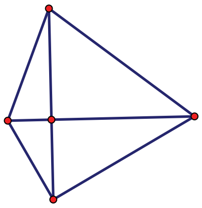

Fig. 9

Through mathematics games, students' confusion in knowledge points can be cleared, and their initiative and enthusiasm in mathematics learning can also be aroused, allowing them to gain mastery of the knowledge points.

5. Fostering thinking through mathematics in daily life

Question 5: Fig. 10 shows a rectangular cake. How can you cut it into two pieces with an equal area?

Fig. 10

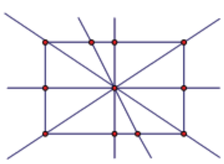

Fig. 11

Judged from the perspective that a rectangle is an axis- and central-symmetric figure, there are countless ways to cut the cake, namely, along the two diagonals or along the straight line connecting the central points of two opposite sides. As depicted in Fig. 11, it can be cut along any straight line beyond the center of the rectangle.

Question 6: Fig. 12 shows a cake comprising two rectangles. How can you cut it into two pieces with an equal area?

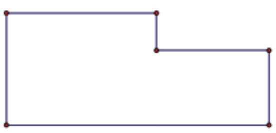

Fig. 12

This problem appears in some after-school tutorials of mathematics. Auxiliary lines are drawn to form two rectangles. The centers of the two rectangles are located and then connected to form a straight line, thereby cutting the cake into two parts with an equal area, as shown in Figs. 13 and 14.

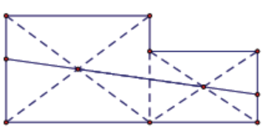

Fig. 13

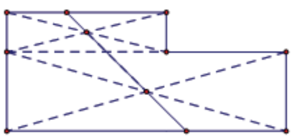

Fig. 14

Question 7: As shown in Fig. 15, a father baked a cake in a rectangular pan as a dessert for his two daughters after they went school. Before the daughters came home, his wife randomly cut a small rectangular piece of cake. Now, how can the father cut the remaining cake into two equally large pieces(3)?

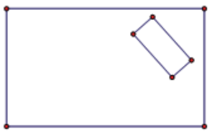

Fig. 15

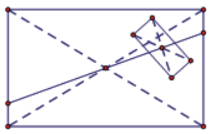

Fig. 16

Unlike Question 6, a rectangular piece was cut from inside the bigger rectangle in this example. Based on the experience of Question 6, the ways to cut the cake were determined. The central points of the two rectangles were identified and then connected to form a straight line, cutting the cake into two equally big pieces. The two trapezoids on both sides of the oblique line were identical in shape, and the smaller rectangle was also divided into two parts with an identical shape. When an equal quantity was subtracted from two equal quantities, the differences were identical; thus, the two daughters can get the same-sized cake piece.

Based on the aforementioned three Question, it can be deduced that a cake can be cut into equal pieces by using countless ways, two ways, and a single way; however, the logic remains the same, namely, the nature of central symmetry, showing the harmonious beauty of mathematics. This problem can be extended to two figures as long as they have a central symmetry, for example, a parallelogram at the outside and a circle inside. They can also be cut in the same way, as shown in Fig. 17.

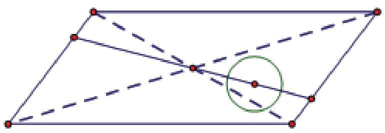

Fig. 17

Exploration in mathematics has deepened our understanding of central symmetry.

6. Relaxing the mind with mathematics snippets

The Pythagorean Theorem is a fundamental theorem taught during Junior High School, and it has long been regarded as the basic principle and a typical example of symbolic-graphic combination, playing an important role in mathematics learning. How to study the Pythagorean Theorem and master the common methods of proving it? The comic, "The Great Sacrifice of a Hundred Bulls—The Adventures of Little Glasses, " written by Professor Li Yupei from the Capital Normal University in the 1920s, a famous popular science writer, is a piece a time travel adventure. It narrates the story of a middle school student who encounters the ancient mathematician Pythagoras, leading to an incident related to the Pythagorean Theorem. This story was adapted into a skit performance "Xiao Qiang's time travel experience–the history of Pythagorean Theorem, " which introduces the controversy related to who on earth discovered the Pythagorean Theorem and the three methods of proving it. In the class, three students were asked to perform the skit. Afterwards, the article was published in the journal *Middle School Mathematics*. The editor commented, "it is an excellent attempt of teaching through lively activities, where the skit performance favored by the students is adopted to explain the history of Pythagorean Theorem." In this way, the students could master the knowledge in an easy and fun way.

As a teacher, the author realizes that mathematics not only involves solving conventional and funny problems, but is also a means to encourage teachers for adjusting the teaching methods flexibly tailored to each student's requirements and enhance their interest in mathematics learning. Overall, mathematics learning can foster the initiative and enthusiasm among students, making learning a fun unlike usual boring learning plans. Effective teaching improves students' academic performance in mathematics and solidifying their learning the highest extent, eventually leading to improved scores and fostering innovation consciousness among students. Students' core literacy and competence improve considerable when teachers employ the most effective methods to teach students about how to observe the real world with a mathematical vision, reflect on the real world with a mathematical mindset, and express the real world with mathematical expressions.

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[3] Authored by Francis Su, translated by Shen Ji'er and Han Xiaoxiao. Strength of Mathematics (M). CITIC Press Group, 2022.

Introduction: Wei Luo (1975-), male, Han nationality, born in Xuzhou, Jiangsu Province, is a first-level primary and secondary school teacher, with a master's degree in science. His research interests are in the field of middle school mathematics education.